

# Compressibility and equation of state of finite nuclei

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We present a new approach for calculating the nuclear equation of state and compressibility for finite nuclei using the density-constrained Hartree-Fock method.

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## I. INTRODUCTION

The study of the nuclear equation of state (EOS) and the behavior of nuclear matter under extreme conditions is crucial to our understanding of many nuclear and astrophysical phenomena. With the increasing availability of radioactive ion-beams [1] the study of structure and reactions of exotic nuclei are now possible, thus providing information concerning the isospin dependence of asymmetric nuclear matter [2, 3]. Recently, the study of the symmetry energy for such systems has been an active area of interest [2, 4, 5, 6].

Most studies of the EOS involve infinite or semi-infinite nuclear matter and examine the dependence of the EOS on the parametrizations of the effective interaction as well as its relation to the macroscopic and microscopic-microscopic models of nuclear matter [7]. For finite nuclei the EOS near the equilibrium density can be investigated via collective observables such as the isoscalar monopole vibrations (breathing mode) [8]. In addition, there are empirical methods relating the compressibility of finite nuclei to that of infinite nuclear matter [9]. However, the behavior of the EOS for finite nuclei far from the equilibrium density is poorly known.

In this manuscript we introduce a new method for calculating the zero temperature EOS and related quantities for finite nuclei within the mean-field description of nuclear properties. In Section II we outline the general formalism for our calculations. Section III discusses the application of the formalism to a sample set of nuclei and the results obtained. The paper is concluded with a summary in Section IV.

## II. FORMALISM

In order to study finite nuclei away from their ground state equilibrium we take advantage of the density constrained Hartree-Fock (DCHF) method [10, 11]. The *density constraint* is a novel numerical method that was developed in the mid 1980's and was used to provide a microscopic description of the formation of shape resonances in light systems [11]. Recently, we have used the same method to calculate heavy-ion interaction potentials from the TDHF time-evolution of nuclear collisions [12]. In the traditional constrained Hartree-Fock (CHF) notation, density constraint corresponds to the

replacement

$$\lambda \hat{Q} \longrightarrow \lambda \hat{p} . \quad (1)$$

The numerical procedure for implementing this constraint and the method for steering the solution towards a specified  $\rho_0(\mathbf{r})$  is discussed in Refs. [10, 11]. The convergence property is as good if not better than the traditional CHF calculations with a constraint on a single collective degree of freedom.

In practice, we have obtained accurate densities for spherical nuclei using a radial Hartree-Fock program. This density was then fitted to a parametric function of the form

$$\rho(r) = \frac{a_0(1 + a_1 r)}{1 + e^{(r-a_2)/a_3}} + \frac{a_4}{1 + e^{(r-a_5)/a_6}} , \quad (2)$$

where  $a_0, \dots, a_6$  denote the parameters to be fitted to reproduce a particular density profile. In all cases the resulting non-linear fits were indistinguishable from the fitted density. Subsequently, the constraining density was obtained via a scale transformation of the above density profile

$$\rho(r) \longrightarrow \rho(sr) , \quad (3)$$

followed by a renormalization to produce the correct mass number for the nucleus under study. This scaling results in the compression of the bulk and stretching of the surface [8]. Naturally, the minimum of all EOS curves occur at the unconstrained Hartree-Fock minimum corresponding to  $s = 1$ . The value of the scale  $s$  was generally limited to the range (0.8, 1.2). In Fig. 1 we show the change in the density for various values of the scaling parameter  $s$  in the case of the  $^{48}\text{Ca}$  nucleus. As the primary interaction we have used the Skyrme SLy4 force [13], with and without the Coulomb term, and including all of the spin-dependent terms. We have performed the calculations using our new Hartree-Fock program discussed in Ref. [14].

In dealing with a finite nucleus we have faced a conceptual problem of deciding what density value to use for plotting the density dependence of the EOS. Unless otherwise stated we have used the *central density* as the reference density value for each value of the scaling parameter  $s$ . Alternatively, one can choose the nuclear matter equilibrium density,  $0.16 \text{ fm}^{-3}$ , as the reference density from the density profile. The calculated values for incompressibility is slightly dependent on the choice of the

reference density point due to the structure in the density profile. Our calculations show that this is about 10% or less and becomes negligible for heavy systems. One disadvantage of using the central density as the reference value is that the equilibrium densities for different nuclei do not occur at the same value, thus making the calculation of the symmetry energy, which is essentially taking the difference of two EOS curves along an isotope chain, erroneous. For this reason, in the calculation of the symmetry energy we have used the density value  $0.16 \text{ fm}^{-3}$  as the equilibrium density.

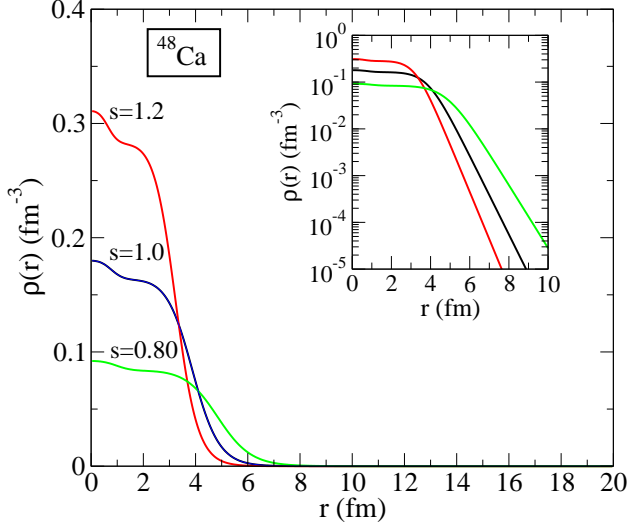


FIG. 1: (Color online) Scaled density obtained for the  $^{48}\text{Ca}$  nucleus for various values of the scaling parameter  $s$ . The inset shows the same quantity on a logarithmic scale. The force used was SLy4 without the Coulomb contribution.

In order to extract the incompressibility coefficient,  $K_A$ , we have expanded the EOS (binding energy per particle as a function of density) around the equilibrium density  $\rho_0$  using the expression

$$\frac{E(\rho)}{A} = \frac{E_0}{A} + \frac{K_A}{18\rho_0^2}(\rho - \rho_0)^2 + \dots, \quad (4)$$

where  $E_0$  is the binding energy at the equilibrium density, and  $K_A$  is the incompressibility coefficient

$$K_A = 9\rho_0^2 \left. \frac{\partial^2(E/A)}{\partial \rho^2} \right|_{\rho=\rho_0}. \quad (5)$$

We found in practice that this expression provides an excellent fit except at the extreme values of the density. To fit the entire curve perfectly a small linear contribution as well as a cubic term could be included. The results for the incompressibility is approximately 5%-10% higher if the full curve is used in the fit. Finally, to extract the symmetry energy we use the expression

$$\mathcal{E}_{\text{sym}}(\rho) = \frac{E(\rho, \alpha)}{A} - \frac{E(\rho, 0)}{A}, \quad (6)$$

where the isospin asymmetry parameter is defined as  $\alpha = (N - Z)/A$ . The  $\alpha$  dependence of the symmetry energy is generally acknowledged to be

$$\mathcal{E}_{\text{sym}}(\rho) = S(\rho)\alpha^2 + O(\alpha^4), \quad (7)$$

where the higher order terms in  $\alpha$  are assumed to be small. Traditionally, the symmetry energy can also be expanded around the equilibrium density as

$$\mathcal{E}_{\text{sym}}(\rho) = \mathcal{E}_{\text{sym}}(\rho_0) + \frac{L}{3\rho_0}(\rho - \rho_0) + \frac{K_{\text{sym}}}{18\rho_0^2}(\rho - \rho_0)^2 + \dots$$

where we have defined the quantities  $L$  and  $K_{\text{sym}}$ , which are related to the symmetry pressure and symmetry compressibility

$$L = 3\rho_0 \left( \frac{\partial \mathcal{E}_{\text{sym}}}{\partial \rho} \right)_{\rho=\rho_0}, \quad K_{\text{sym}} = 9\rho_0^2 \left( \frac{\partial^2 \mathcal{E}_{\text{sym}}}{\partial \rho^2} \right)_{\rho=\rho_0}.$$

### III. NUMERICAL STUDIES

Using our approach we have first investigated the EOS for the  $^{16}\text{O}$  nucleus, with two forces, SLy4 and SkM\* [15], without the Coulomb interaction. We found very little difference between the two forces as expected since most modern Skyrme forces have similar nuclear matter incompressibility values. For the incompressibility we find  $K_A = 116 \text{ MeV}$ . We have then repeated the same two

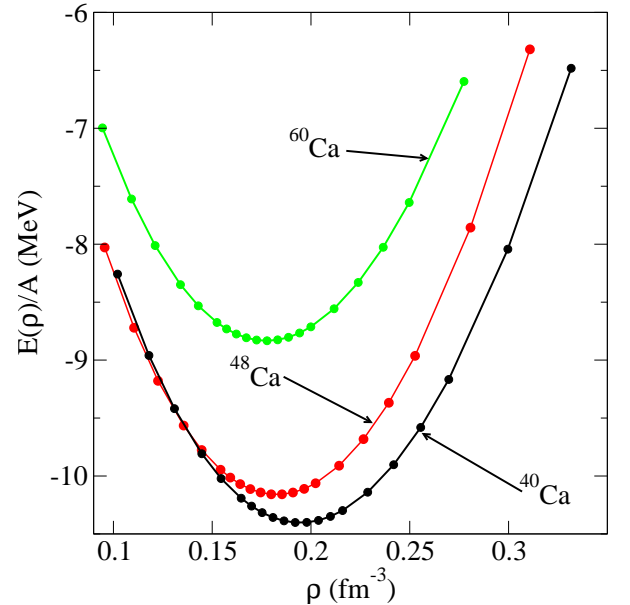


FIG. 2: (Color online) EOS for the  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$ , and  $^{60}\text{Ca}$  nuclei as a function of density. Please see the comments in the manuscript regarding the density values. The force used was SLy4 without the Coulomb contribution.

force study for the  $^{40}\text{Ca}$  nucleus, yielding an incompressibility of  $K_A = 141 \text{ MeV}$ . In order to investigate the

behavior of neutron rich systems we have performed calculations for  $^{48}\text{Ca}$  and  $^{60}\text{Ca}$  systems. In Fig. 2 we show the EOS for all of these Ca nuclei without the Coulomb force. Again, we stress that the density scale shown in Fig. 2 is determined by choosing the central density in the density profile of each nucleus. Consequently, the equilibrium value for the EOS is different for each nucleus. As we can see from Fig. 2, the  $^{40}\text{Ca}$  and  $^{48}\text{Ca}$  systems have a very similar EOS behavior, the primary difference being the energy shift due to the difference in the two binding energies. The calculated incompressibility of  $^{48}\text{Ca}$  is  $K_A = 155$  MeV, only slightly higher than the one for  $^{40}\text{Ca}$ . However, the situation for  $^{60}\text{Ca}$  is significantly different, in addition to the large shift in the energy scale the incompressibility decreases to a value of  $K_A = 136.5$  MeV, indicating a somewhat softer nucleus. The values for the incompressibility obtained here, perhaps with the exception of the  $^{60}\text{Ca}$  system, are in general agreement with those given in Ref. [9].

We have repeated some of the calculations by including the Coulomb interaction as well. In general, EOS curves are shifted up due to the decrease in the binding energy per nucleon. In addition, we see a small decrease in the incompressibility modulus. For  $^{40}\text{Ca}$  we find  $K_A = 138.8$  MeV. The incompressibility for  $^{48}\text{Ca}$  decreases to  $K_A = 147.7$  MeV, and this value drops down to  $K_A = 123.5$  MeV for  $^{60}\text{Ca}$ .

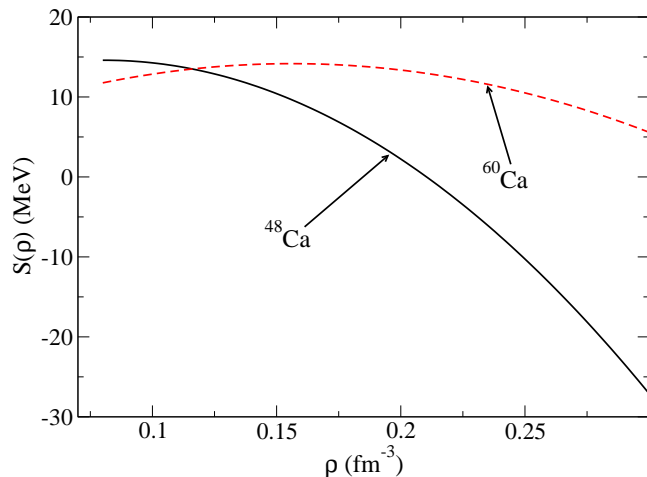


FIG. 3: (Color online) The quantity  $S(\rho)$  for the  $^{48}\text{Ca}$  and  $^{60}\text{Ca}$  nuclei as a function of density.

We have also investigated the symmetry energy obtained from Eq. 6, which is essentially the difference between the curves shown in Fig. 2. The results for the quantity  $S(\rho)$  defined in Eq. 7, corresponding to nuclei  $^{48}\text{Ca}$  and  $^{60}\text{Ca}$  having  $\alpha$  values of 0.16667 and 0.33334, respectively, are shown in Fig. 3. As we observe again, the two systems behave very differently. Unlike infinite nuclear matter the curves do not cross at the equilibrium density since the binding energy per nucleon is different for different nuclei. The values of  $S$  around the equilibrium density are about 10-14 MeV, which is considerably lower than the estimated range for nuclear matter value of 32 MeV for the SLy4 force. Using the results of Fig. 3 we have extracted the quantities  $L$  and  $K_{\text{sym}}$  for  $^{48}\text{Ca}$  and  $^{60}\text{Ca}$ . The results show a very large variation between the two systems. For  $^{48}\text{Ca}$  we find 66.6 MeV and -403 MeV for  $L$  and  $K_{\text{sym}}$ , while for  $^{60}\text{Ca}$  these numbers become 1.55 MeV and -193 MeV, respectively.

#### IV. CONCLUSIONS

We have introduced a new method for calculating the nuclear EOS for finite nuclei including all of the terms in the nuclear effective interaction. The calculated values agree well with known values obtained by other means, such as those deduced from experimental giant monopole resonances. Much work has gone into understanding the effects of various components of the effective interaction which vanish or become very small in the infinite nuclear matter limit. With the increased availability of new radioactive neutron and proton rich nuclei the study of EOS and symmetry energy along isotope chains of finite nuclei have become more urgent. We believe that the density constrained HF method is a step in this direction.

#### Acknowledgments

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